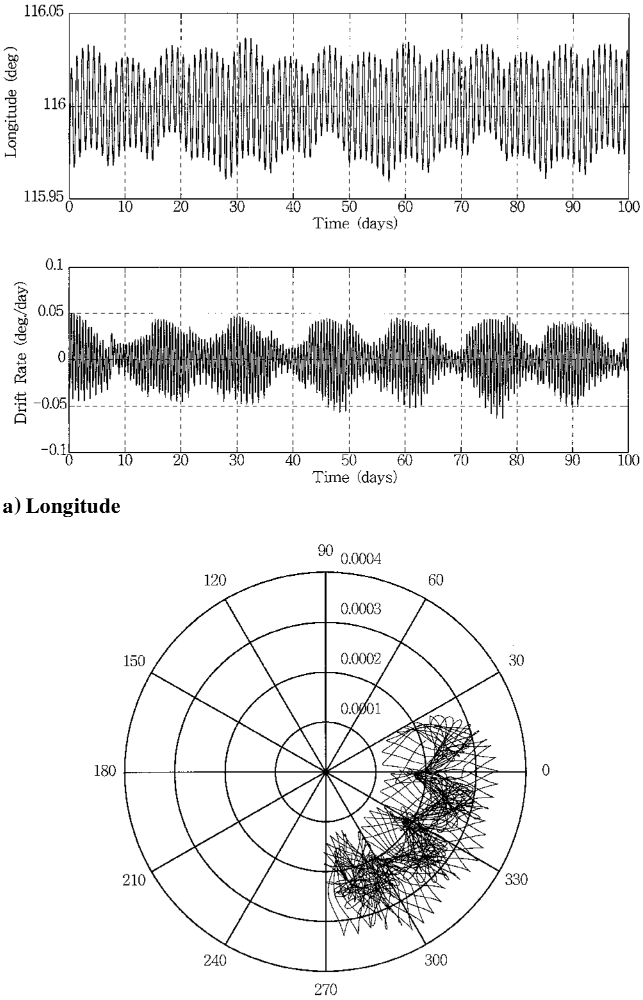
Table 1 Parameters used in the simulation

Nominal longitude λ_s	116°E
Station-keeping deadband	±0.05 deg
Station-keeping cycle T_{EW}	7 days
Spacecraft area-to-mass ratio	0.024 m²/kg
Maximum eccentricity e_c	0.000235
Sun lag angle α_{EW}	3.97 deg



a) Longitude

b) Eccentricity vector

Fig. 3 Nonlinear simulation results.

Figure 3 displays the time history of longitude drift and the eccentricity vector for approximately 100 days. It is clear that the spacecraft is effectively maintained near the nominal station and the growth of the eccentricity is well under control.

Conclusions

In this Note, a predictive targeting strategy for E/W station keeping of GEO spacecrafts was presented and its performance verified in the nonlinear simulations. The important difference from the previous methods is that the spacecraft starts its drifting toward the target conditions at the next station-keeping cycle, not from the conditions at the current cycle. Also, the longitude angle itself instead of drift rate is used in obtaining the target conditions. As a result, the spacecraft longitude could be controlled in a more precise and robust manner in the presence of modeling or maneuver execution errors.

Acknowledgment

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Robust Fault Diagnosis for a Class of Linear Systems with Uncertainty

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I. Introduction

THE process of detection and isolation of system faults has been of considerable interest during the past two decades. Fruitful results can be found in several excellent survey papers.[1–6] Research is still under way into the development of more effective solutions for fault detection and isolation (FDI) in automatic control systems.

The model-based FDI concept is always under a number of idealized assumptions. Among them is that the mathematical model of the process is exactly known. In practice, this assumption can never be perfectly satisfied, as an accurate mathematical model of a physical process is never available. Because of this, there is often a mismatch between the actual process and its mathematical model, even if no fault in the process occurs. Such a mismatch causes fundamental methodological difficulties in FDI applications. They are a source of false alarms that can corrupt the performance of the FDI system.

To overcome the difficulty, FDI systems have to be made robust, i.e., insensitive or even invariant to such modeling errors or disturbances. A system designed to provide both sensitivity to faults and robustness to modeling error or disturbances is called a robust FDI scheme.[7,8] One of the most successful robust fault diagnosis approaches is the use of the disturbance decoupling principle. This can be done by use of unknown input observers.[9–11] But in some cases, such as unstructured uncertainties in the system, perfect decoupling is not possible, so robust observers are to be designed for FDI.

In this Note we consider the FDI problem for a class of linear systems with uncertainties. At first, the system is transformed into two subsystems. The first subsystem is decoupled from actuator fault and the other is affected by the fault, but its states can be measured directly. As a generalization of the observer design approach in Ref. 12, a robust observer design is proposed for FDI. By using the estimation of states with bounded accuracy, we can approximate

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the fault by using the discretization of the other subsystems. When fault signal is greater than certain threshold, one can detect it or even isolate it.

This Note is organized as follows. Section II gives conditions for the existence of a transformation that converts the original uncertain system into a desired form. Then a robust observer design is given based on the transformed system. In Sec. III, fault detection and isolation are discussed. An application example is included in Sec. IV, followed by some concluding remarks in Sec. V.

II. Robust Observer Design

Consider the following system with uncertainty,

$$\dot{x} = Ax + Bu + Ef_a + D\xi(x) \tag{1}$$

$$y = Cx = [C_1 \quad 0]x \tag{2}$$

where the state is $x \in R^n$, the input is $u \in R^m$, and the output is $y \in R^r$. The fault is modeled as an additional disturbance input $f_a \in R^q$, with $q < r$, and $\xi(x)$ with $\xi(0) = 0$ model system uncertainties, high-order nonlinearities, and disturbances. C_1 is an $r \times r$ nonsingular matrix.

Assumption 1: $\mathrm{Rank}(CE) = q$.

Define a transformation $x = N^{-1}z$, where

$$N = \begin{bmatrix} C_1 & 0 \\ 0 & I \end{bmatrix}$$

Then the system described by Eqs. (1) and (2) can be transformed into

$$\dot{z} = \bar{A}z + \bar{B}u + \bar{E}f + \bar{D}\xi$$

$$= \begin{bmatrix} \bar{A}_1 \\ \bar{A}_2 \\ \bar{A}_3 \end{bmatrix} z + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ \bar{B}_3 \end{bmatrix} u + \begin{bmatrix} \bar{E}_1 \\ \bar{E}_2 \\ \bar{E}_3 \end{bmatrix} f_a + \begin{bmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \end{bmatrix} \xi \tag{3}$$

$$y = \bar{C}z = \begin{bmatrix} I_{(r-q)\times(r-q)} & 0 & 0 \\ 0 & I_q & 0 \end{bmatrix} z \tag{4}$$

where

$$\bar{A} = NAN^{-1}, \qquad \bar{B} = NB, \qquad \bar{E} = NE, \qquad \bar{D} = ND$$

Partition z is given as

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ z_3 \end{bmatrix}$$

where $z_3 \in R^{n-r}$, whose estimate is required.

According to Assumption 1, $\mathrm{rank}(\bar{C}\bar{E}) = q$. From the structure of matrix $\bar{C}$, it is easy to show that

$$\mathrm{rank}\begin{bmatrix} \bar{E}_1 \\ \bar{E}_2 \end{bmatrix} = q \tag{5}$$

From Eq. (5), without loss of any generality, it can be assumed that $\bar{E}_2$ is nonsingular.

Assumption 2: There exist positive constants α and β such that

$$\left\| \left(\bar{D}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{D}_2\right)\xi \right\| \le \alpha, \qquad \bar{D}_1 - \bar{E}_1\bar{E}_2^{-1}\bar{D}_2 = 0$$

$$\|(\bar{D}_2)\xi\| \le \beta$$

Remark 1: Assumption 2 implies the partial decoupling of the system uncertainty in the transformed system (3–4). This improves existing results in, e.g., Refs. 9 and 10, in which a perfect decoupling of system uncertainty is required.

Let

$$S = \begin{bmatrix} I & -\bar{E}_1\bar{E}_2^{-1} & 0 \\ 0 & I & 0 \\ 0 & -\bar{E}_3\bar{E}_2^{-1} & I \end{bmatrix} \tag{6}$$

By premultiplying Eq. (6) into Eq. (3), we have that

$$\begin{bmatrix} \dot{y}_1 - \bar{E}_1\bar{E}_2^{-1}\dot{y}_2 \\ \dot{y}_2 \\ \dot{z}_3 - \bar{E}_3\bar{E}_2^{-1}\dot{y}_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_1 - \bar{E}_1\bar{E}_2^{-1}\bar{A}_2 \\ \bar{A}_2 \\ \bar{A}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{A}_2 \end{bmatrix} z$$

$$+ \begin{bmatrix} \bar{B}_1 - \bar{E}_1\bar{E}_2^{-1}\bar{B}_2 \\ \bar{B}_2 \\ \bar{B}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{B}_2 \end{bmatrix} u + \begin{bmatrix} 0 \\ \bar{E}_2 \\ 0 \end{bmatrix} f_a + \begin{bmatrix} 0 \\ \bar{D}_2 \\ \bar{D}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{D}_2 \end{bmatrix} \xi \tag{7}$$

It is clear that in matrix (7) the actuator faults enter only through the second block row, whereas the other two block rows are not affected by any faults. Define

$$G_i = \bar{A}_i - \bar{E}_i\bar{E}_2^{-1}\bar{A}_2, \qquad H_i = \bar{B}_i - \bar{E}_i\bar{E}_2^{-1}\bar{B}_2, \qquad i = 1, 3$$

Then the first and the third block rows of matrix (7) can be rewritten as

$$\dot{y}_1 - \bar{E}_1\bar{E}_2^{-1}\dot{y}_2 = G_1z + H_1u \tag{8}$$

$$\dot{z}_3 - \bar{E}_3\bar{E}_2^{-1}\dot{y}_2 = G_3z + H_3u + \left(\bar{D}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{D}_2\right)\xi \tag{9}$$

Partitioning G_i as

$$G_i = [G_{i1} \quad G_{i2} \quad G_{i3}], \qquad i = 1, 3$$

then we can rewrite Eqs. (8) and (9) as

$$\dot{z}_3 = G_{33}z_3 + s + \left(\bar{D}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{D}_2\right)\xi \tag{10}$$

$$v = G_{13}z_3 \tag{11}$$

where

$$s = G_{31}y_1 + G_{32}y_2 + \bar{E}_3\bar{E}_2^{-1}\dot{y}_2 + H_3u \tag{12}$$

$$v = \dot{y}_1 - \bar{E}_1\bar{E}_2^{-1}\dot{y}_2 - G_{11}y_1 - G_{12}y_2 - H_1u \tag{13}$$

Since the dynamical system represented by Eqs. (10) and (11) is driven by known input s and uncertain term $(\bar{D}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{D}_2)\xi$, its state can be estimated with a robust observer:

$$\dot{\hat{z}}_3 = G_{33}\hat{z}_3 + s + K(v - G_{13}\hat{z}_3) \tag{14}$$

where K is the observer's gain. Substituting v and s of Eqs. (12) and (13) into Eq. (14) yields

$$\dot{\hat{z}}_3 = (G_{33} - KG_{13})\hat{z}_3 + (G_{31} - KG_{11})y_1 + (G_{32} - KG_{12})y_2$$

$$+ (H_3 - KH_1)u + \left(\bar{E}_3\bar{E}_2^{-1} - K\bar{E}_1\bar{E}_2^{-1}\right)\dot{y}_2 + K\dot{y}_1 \tag{15}$$

Note that the above observer uses the derivative of the outputs that is not available for direct measurement. To solve this problem a new variable W is defined as follows, in order to eliminate the need for differentiating the output:

$$W = \hat{z}_3 - \left[\left(\bar{E}_3\bar{E}_2^{-1} - K\bar{E}_1\bar{E}_2^{-1}\right)y_2 + Ky_1\right] \tag{16}$$

Then Eq. (15) can be expressed in the following form:

$$\dot{W} = (G_{33} - KG_{13})W + (H_3 - KH_1)u$$

$$+ [G_{31} - KG_{11} + K(G_{33} - KG_{13})]y_1$$

$$+ \left[G_{32} - KG_{12} + (G_{33} - KG_{13})\left(\bar{E}_3\bar{E}_2^{-1} - K\bar{E}_1\bar{E}_2^{-1}\right)\right]y_2 \tag{17}$$

From Eqs. (10), (11), and (14), the error dynamics of $\tilde{z}_3 = z_3 - \hat{z}_3$ is given by

$$\dot{\tilde{z}}_3 = (G_{33} - KG_{13})\tilde{z}_3 + \left(\bar{D}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{D}_2\right)\xi \tag{18}$$

The following theorem establishes conditions under which the observer's error dynamics (18) converges to a bounded set.

Theorem 1: If the pair $\{G_{33}, G_{13}\}$ is observable and Assumptions 1 and 2 hold, then system (10) can be estimated by Eqs. (16) and (17). Furthermore,

$$\|\tilde{z}_3(t)\| \leq \frac{2\alpha[\lambda_{\max}(P)]^2}{\lambda_{\min}(Q)\lambda_{\min}(P)} \triangleq \gamma \tag{19}$$

for all $t \geq 0$, where $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote, respectively, the maximum and minimum eigenvalues of a symmetric matrix and P is the positive definite solution of

$$P(G_{33} - KG_{13}) + (G_{33} - KG_{13})^T P = -Q \tag{20}$$

where Q is positive definite.

Proof: Consider the Lyapunov function

$$V = \tilde{z}_3^T P \tilde{z}_3 \tag{21}$$

where P is the positive definite solution of Eq. (20). From error dynamics (18) and Assumption 2, we have that

$$\begin{aligned}\dot{V} &= -\tilde{z}_3^T Q\tilde{z}_3 + 2\tilde{z}_3 P\left(\bar{D}_3 - \bar{E}_3\bar{E}_2^{-1}\bar{D}_2\right)\xi \\ &\leq -\lambda_{\min}(Q)\|\tilde{z}_3\|^2 + 2\alpha\|P\|\|\tilde{z}_3\|\end{aligned} \tag{22}$$

Noting that

$$\lambda_{\min}(P)\|\tilde{z}_3\|^2 \leq V \leq \lambda_{\max}(P)\|\tilde{z}_3\|^2$$

one can further get

$$V \leq \frac{4\alpha^2[\lambda_{\max}(P)]^4}{\lambda_{\min}(P)[\lambda_{\min}(Q)]^2} \tag{23}$$

Thus relation (19) holds. This completes the proof. ∎

Remark 2: Let $\bar{G} = G_{33} - KG_{13}$; from relation (19), Eq. (20), and Theorem 2.4 in Ref. 13, we have

$$\|\tilde{z}_3(t)\| \leq \frac{2\alpha}{|\lambda_{\max}(\bar{G} + \bar{G}^T)|}\left[\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)}\right]^2 \tag{24}$$

Because the pair $\{G_{33}, G_{13}\}$ is observable, $|\lambda_{\max}(\bar{G} + \bar{G}^T)|$ can in theory be made arbitrarily large if an appropriate observer gain K is chosen. Hence, the above estimate error $\tilde{z}$ can be made arbitrarily small. Of course, because of sensor noise in practice, this observer gain K cannot be too large either.

Remark 3: If there is no uncertainty in the system, that is, $\alpha = 0$, $\beta = 0$, the obtained result in Theorem 1 is similar to that in Ref. 12.

III. Fault Detection and Isolation

In this section, a simple approach for detecting and isolating the actuator fault is proposed to estimate the magnitude of the actuator fault f_a.

Theorem 2: Suppose that the magnitude of the actuator fault signal satisfies

$$\|f_a\| > \left\|\bar{E}_2^{-1}\bar{A}_{23}\right\|\gamma + \left\|\bar{E}_2^{-1}\right\|\beta \tag{25}$$

where γ is as in relation (19), α is as in Assumption 2, and $\bar{A}_{23}$ is the third block column of $\bar{A}_2$ corresponding to z_3 in Eq. (4). Then, under the assumptions in Theorem 1, actuator faults can be detected.

Proof: The second row within matrix (7) is written as

$$\dot{y}_2 = \bar{A}_{21}y_1 + \bar{A}_{22}y_2 + \bar{A}_{23}z_3 + \bar{B}_2 u + \bar{E}_2 f_a + \bar{D}_2\xi \tag{26}$$

Discretization of this system yields

$$\begin{aligned}T\bar{E}_2 f_a(k) &= y_2(k+1) - y_2(k) - T[\bar{A}_{21}y_1(k) \\ &+ \bar{A}_{22}y_2(k) + \bar{A}_{23}z_3(k)] + T\bar{D}_2\xi\end{aligned} \tag{27}$$

where k represents the kth time step and T is the sampling period. Assuming that no fault occurs during the initial transient of the observer and using the estimation $\hat{z}_3(k)$ for $z_3(k)$, we can approximate the actuator fault as

$$\begin{aligned}\hat{f}_a(k) = {}& \bar{E}_2^{-1} \\ &\times\left[\frac{y_2(k+1) - y_2(k)}{T} - \bar{A}_{21}y_1(k) - \bar{A}_{22}y_2(k) - \bar{A}_{23}\hat{z}_3(k)\right]\end{aligned} \tag{28}$$

With no uncertainty, the actuator fault estimate $\hat{f}_a$ would have a zero norm if there is no actuator fault and a nonzero norm if an actuator fault occurs. The estimate error of the actuator fault is given by

$$\tilde{f}_a(k) = \hat{f}_a(k) - f_a(k) = -\bar{E}_2^{-1}\bar{A}_{23}\tilde{z}_3 + \bar{E}_2^{-1}\bar{D}_2\xi \tag{29}$$

which, with relation (19) and Assumption 2, is bounded by

$$\|\tilde{f}_a(k)\| \leq \left\|\bar{E}_2^{-1}\bar{A}_{23}\right\|\gamma + \left\|\bar{E}_2^{-1}\right\|\beta \tag{30}$$

If this norm bound of the fault estimate error is taken as the threshold, actuator fault detection can be achieved if condition (25) holds. In addition, the fault can be isolated by checking the nonzero entry or orientation of $\hat{f}_a$. ∎

IV. Example

To illustrate the proposed method, the F16XL example in Refs. 14 and 15 is examined. The aircraft dynamics is linearized about trimmed level flight at a 10,000-ft altitude. For simplicity, a reduced-order five-state model of the longitudinal dynamics is considered:

$$\dot{x} = Ax + E\mu_{A_z} + D\mu_{w_g} \tag{31}$$

$$y = Cx \tag{32}$$

where $x = [u \;\; w \;\; q \;\; \theta \;\; w_g]^T$ is a state in which u is the forward velocity (in feet per second), w the vertical velocity (in feet per second), q is the pitch rate (in degree per second), θ is the pitch (in degrees), and w_g is the wind gust (in feet per second); μ_{A_z} denotes a normal accelerometer fault, and μ_{w_g} is the wind-gust input that is a zero-mean white-noise process with unit spectral density. The measurement outputs are $y = [q \;\; \theta \;\; A_z \;\; A_x]^T$, where q is the pitch rate (in radians per second), θ is the pitch angle (in radians), A_z is the normal acceleration (in feet per square second), and A_x is the forward acceleration (in feet per square second). The system matrices are

$$A = \begin{bmatrix} -0.0674 & 0.0430 & -0.8886 & -0.5587 & 0.0430 \\ 0.0205 & -1.4666 & 16.5800 & -0.0299 & -1.4666 \\ 0.1377 & -1.6788 & -0.6819 & 0 & -1.6788 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.1948 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.6003 & 0 \\ 0.9429 & -1.3706 \\ 0 & -1.5003 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2.0156 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.0139 & 1.0517 & 0.1485 & -0.0299 & 0 \\ -0.0677 & 0.0431 & 0.0171 & 0 & 0 \end{bmatrix}$$

By selecting

$$N = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.0139 & 1.0517 & 0.1485 & -0.0299 & 0 \\ -0.0677 & 0.0431 & 0.0171 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

we have that

$$\bar{A} = NAN^{-1} =$$

$$\begin{bmatrix} -0.4191 & -0.04486 & 1.500 & -2.342 & -1.679 \\ 1 & 0 & 0 & 0 & 0 \\ 17.56 & -0.08897 & 1.664 & -0.9484 & -1.791 \\ 0.7779 & -0.0340 & -0.08473 & -0.1326 & -0.09483 \\ 0 & 0 & 0 & 0 & -1.195 \end{bmatrix}$$

$$\bar{E} = NE = \begin{bmatrix} 0 & -1.50 \\ 0 & 0 \\ 1 & -1.664 \\ -0.132 \times 10^{-5} & -0.08473 \\ 0 & 0 \end{bmatrix}$$

$$\bar{D} = ND = [0 \quad 0 \quad 0 \quad 0 \quad 2.016]^T$$

$$\bar{D}_1 = 0, \qquad \bar{D}_2 = 0, \qquad \beta = 0$$

$$G_1 = \bar{A}_1 - \bar{E}_1 \bar{E}_2^{-1} \bar{A}_2 =$$

$$\begin{bmatrix} 14.19 & -0.6469 & 0.7837 \times 10^{-4} & 0.6445 \times 10^{-2} & 0.3602 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G_3 = \bar{A}_3 - \bar{E}_3 \bar{E}_2^{-1} \bar{A}_2 = [0 \quad 0 \quad 0 \quad 0 \quad -1.195]$$

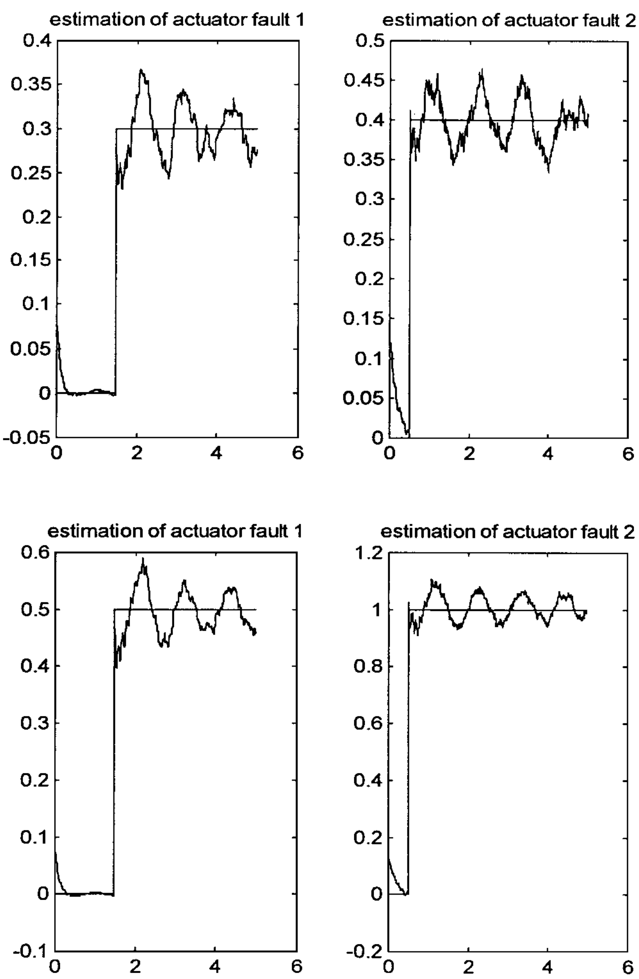

Fig. 1 Fault identification and estimation.

Thus $(G_{33} \;\; G_{13})$ is observable, and all the assumptions in Theorem 2 hold. Let $K = [k_1 \;\; k_2]$, according to Theorem 2; the normal accelerometer fault μ_{A_z} can be detected if

$$\|\mu_{A_z}\| \geq \frac{3.79}{0.3602k_1 + 1.195} \tag{33}$$

In the simulation, there is sensor noise (of variance 0.3) corrupting the system measurements; the gain k_1 is selected to be 50. The threshold is 0.19. Figure 1 illustrates the estimates of actuator faults 1 and 2 as follows:

$$f_{a1} = \begin{cases} 0 & \text{for} \quad 0 \leq t \leq 1.5 \\ 0.3 & \text{for} \quad 1.5 < t \leq 5 \end{cases}$$

$$f_{a2} = \begin{cases} 0 & \text{for} \quad 0 \leq t \leq 0.5 \\ 0.4 & \text{for} \quad 0.5 < t \leq 5 \end{cases}$$

$$f_{a1} = \begin{cases} 0 & \text{for} \quad 0 \leq t \leq 1.5 \\ 0.5 & \text{for} \quad 1.5 < t \leq 5 \end{cases}$$

$$f_{a2} = \begin{cases} 0 & \text{for} \quad 0 \leq t \leq 0.5 \\ 1 & \text{for} \quad 0.5 < t \leq 5 \end{cases}$$

It is shown that the faults estimation and isolation are in good performance despite the disturbance in the system.

Remark 4: In Ref. 14, a left-eigenvector assignment approach is developed that yields improved eigenvector conditioning and sensitivity to system parameter variations. As shown in that paper, it is not easy to search for a best set of such useful eigenvectors. The method proposed in this Note has two advantages. One is that it can detect not only occurrences but also amplitudes of faults. The other is that it can be used for a system with uncertainties that cannot be decoupled from faults or residuals.

V. Conclusion

The fault diagnostic approach in this Note uses a robust observer to detect and identify actuator faults with certain accuracy under bounded uncertainties. An aircraft example is given to illustrate the proposed scheme. A similar method can be used for detection and isolation of sensor faults. By using robust or adaptive observers, FDI for nonlinear systems with uncertainties will be investigated in future.

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Band-Limited Actuator and Sensor Selection for Disturbance Rejection

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Introduction

A METHOD of selecting actuator and sensor locations from a predetermined set of candidate locations based on the Hankel singular values of the controllability and observability Gramian of flexible structures was outlined previously.[1–4] Within the original formulation, the actuator/sensor selection methodology was based entirely on the Hankel singular values of the control path (P_{yu}) for some finite set of modes and some predetermined set of actuators and sensors.[1,2] Later revisions to this method have included a weighting of the Hankel singular values of the control path (P_{yu}) by that of the Hankel singular values of the performance path (P_{zw}) (Refs. 3 and 4). In the most recent application, the disturbance rejection approach to actuator/sensor selection was applied to the piezoceramic aeroelastic response tailoring investigation wind-tunnel model at the NASA Langley Research Center.[4] The results of that study demonstrated a significant advantage to the application of the disturbance rejection approach.

The methodology outlined herein serves as an extension to the disturbance rejection approach with an added metric aimed at robustness with respect to out-of-bandwidth response. Although the method proposed by Lim[3] determines the actuators and sensors that couple best to modes present in the disturbance path, there is no penalty associated with coupling to higher-order modes that are out of the desired bandwidth for control. All practical realizations of control systems are implemented over some finite bandwidth, and for digital realizations, bandwidth limitations are typically imposed by the computational speed of the digital signal processor and the number of modes within the desired bandwidth. Accurate models of the dynamics within the desired bandwidth for control are required; however, model fidelity typically suffers beyond the identified bandwidth. This is particularly true for distributed parameter systems that have infinite dimensional theoretical models. As a result, compensators that incorporate a roll-off are desired and low-pass filters are frequently used in practice to attenuate the response at higher frequency. The response of out-of-bandwidth modes can lead to spillover and stability issues, particularly in the presence of "aggressive" controllers. However, the best method of attenuating the response out of the desired bandwidth is to have poor observability and/or controllability of such modes as an intrinsic property of the open-loop plant. The proposed approach thus serves to simplify robust controller design in the mitigation of spillover.

To this end, an additional metric is introduced for the selection of actuators and sensors for disturbance rejection. Within the bandwidth of desired performance, a metric consistent with that outlined by Lim[3] is computed to determine the actuators and sensors that couple best for disturbance rejection. However, out of the desired bandwidth, a metric is introduced that penalizes the coupling of actuators and sensors to out-of-bandwidth modes. A combination of the two metrics leads to a tradeoff between performance and robustness to spillover effects in the actuator/sensor selection methodology.

The methodology proposed is outlined in the subsequent section. Following the introduction of the new metric, an analytical example based on a structural acoustic control problem is presented. Conclusions are drawn from the results.

Band-Limited Placement Metric for Disturbance Rejection

The problem of interest is the control of a standard two-port system, as shown in Fig. 1. The upper transfer function $P_{zw}(s)$ represents the path from disturbances $w(s)$ to a measure of the closed-loop system performance $z(s)$. This path is determined by the definition of the active control problem, with tradeoffs that reflect available resources, e.g., control energy, or robustness requirements. The lower transfer function $P_{yu}(s)$ represents the path from the control inputs $u(s)$ to the measured outputs $y(s)$ and is a function of the choice and placement of actuators and sensors, respectively, for the control system. Although usually taken as a given for control design, often there is freedom determining sensor and actuator locations, yielding a design decision that impacts the closed-loop performance. Good placement for control design can be determined from the influence of transducers on the open-loop system's controllability and observability, measured in terms of the Hankel singular values.

As noted by Lim,[3] the Hankel singular values (HSVs) from the disturbance to performance outputs illustrated in Fig. 1 are denoted as

$$\Gamma_{zw}^2 = \mathrm{diag}\left(\gamma_{zw1}^2, \ldots, \gamma_{zwn}^2\right) \tag{1}$$

where Γ_{zw}^2 is a diagonal matrix of the n HSVs of the system. The HSVs provide a measure of the degree of coupling of each of the n modes associated with the performance path of the plant $P_{zw}(s)$. The HSV for each of the pth actuators and qth sensors of the control path can be defined as

$$\Gamma_{y_q u_p}^2 = \mathrm{diag}\left(\gamma_{y_q u_p 1}^2, \ldots, \gamma_{y_q u_p n}^2\right) \tag{2}$$

A baseline reference of HSVs for all p actuators and q sensors is defined as

$$\bar{\Gamma}_{yu}^2 = \mathrm{diag}\left(\bar{\gamma}_{yu1}^2, \ldots, \bar{\gamma}_{yun}^2\right) \tag{3}$$

The placement metric for disturbance rejection, as outlined by Lim,[3] can be expressed as follows for the qth sensor and pth actuator:

$$J_{qp} = \sum_{i=1}^{n} \left(\frac{\gamma_{y_q u_p i}^4}{\bar{\gamma}_{yui}^4}\right) \gamma_{zwi}^4 \tag{4}$$

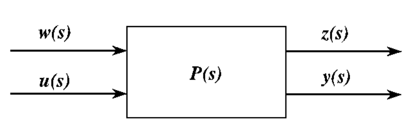

Fig. 1 Block diagram of the two-port design model.

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